Solutions to JEE Advanced Booster Test – 4|JEE 2024

PHYSICS

SECTION-1

Single Choice Type

1.(A) We have,

$$\frac{dQ}{dt} = -KA\frac{d\theta}{dx} = -K\,4\pi r^2\frac{d\theta}{dr}$$

But $\frac{dQ}{dt} = P,$ thermal power

$$\therefore\ P = -4K\pi r^2\frac{d\theta}{dr} \qquad \text{i.e.,}\ \frac{dr}{r^2} = -\frac{4\pi K}{P}d\theta$$

Integrating

$$\int_{r_1}^{r_2}\frac{dr}{r^2} = \frac{-4\pi K}{P}\int_{T_1}^{T_2}d\theta$$

$$\left[-\frac{1}{r}\right]_{r_1}^{r_2} = \frac{-4\pi K}{P}(T_2 - T_1)$$

$$\therefore\ \frac{1}{r_1} - \frac{1}{r_2} = \frac{4\pi K}{P}(T_1 - T_2)$$

$$\therefore\ r_1\ r_2 = \frac{P}{4\pi K}\left[\frac{1}{\frac{(T_1-T_2)}{(r_2-r_1)}}\right],\ \text{numerically}$$

Since, $\frac{T_1 - T_2}{r_2 - r_1} \le 1$(given)

Product of the radii $r_1\ r_2 \ge \frac{P}{4\pi K}$

2.(D) Let V be the volume of liquid flown out of container

$$V = \frac{AL}{3}[\gamma\Delta T + 2\gamma\Delta T + 3\gamma\Delta T] - \left[A(1+\gamma\Delta T)L\left(1+\frac{\gamma}{2}\Delta T\right) - AL\right]$$

$$= 2AL\gamma\Delta T - AL\left(1+\frac{3\gamma}{2}\Delta T\right) + AL = \frac{AL\gamma\Delta T}{2}$$

3.(D) $\pi R^2 2H\rho g + \frac{2}{3}\pi R^3 2\rho g = F$

$$2\pi R^2 H\rho g + \frac{4\pi R^3\rho g}{3} = F$$

$$2\pi R^2\rho g\left(H + \frac{2R}{3}\right) = F$$

4.(A) Minima will be formed at O, if $SS_1 + S_1O - SS_2 - S_2O = \frac{n\lambda}{2}$, $n = 1, 2, ...$

For minimum value of d, $n = 1$

$\therefore\ \frac{\lambda}{4} = \sqrt{1+d^2} - 1$

$(1+d^2)^{1/2} - 1 = \frac{\lambda}{4}$

$1 + \frac{d^2}{2} - 1 = \frac{\lambda}{4}$ (Neglecting smaller terms)

$\Rightarrow\ d = \sqrt{\frac{\lambda}{2}}$

5.(B) For $TIR, r_2 > \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Since, $r_1 + r_2 = 60°$, we get $r_1 < 60° - \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\Rightarrow\ r_1 < \sin^{-1}\left[(\sin 60°)\cos\left(\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)\right) - \frac{1}{\sqrt{3}}\cos 60°\right]$

$\Rightarrow\ r_1 < \sin^{-1}\left[\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{\sqrt{3}} - \frac{1}{\sqrt{3}}\cdot\frac{1}{2}\right] \quad \Rightarrow \quad r_1 < \sin^{-1}\left(\frac{\sqrt{6}-1}{2\sqrt{3}}\right)$

Now, $\sin i = \sqrt{3}\sin r_1 \quad \Rightarrow \quad i < \sin^{-1}\left(\frac{\sqrt{6}-1}{2}\right)$

Multiple Correct Type

6.(AB) $\frac{1}{F} = \frac{(\mu-1)}{R} = \frac{1}{20}$

$\Rightarrow\ 20 = \frac{R}{(\mu-1)} \qquad \ldots$(i)

If equiconvex lens of $f = 20$

Then $\frac{1}{20} = (\mu-1)\frac{2}{R} \qquad \ldots$(ii)

From equation (i) and (ii) we conclude

$\therefore\ R = 40(\mu-1)$

That option (A) is correct

$-\frac{1}{f_L} = (\mu-1)\left[\frac{1}{R} - \frac{1}{\infty}\right] = \frac{(\mu-1)}{R}$ for refraction at the convex surface

For reflection at the silvered plane surface

$F_M = \infty$

$\therefore$ power $P_M = 0$

For refraction at the convex surface again

$$P_L = \frac{(\mu - 1)}{R}$$

Hence power of the system

$$P = P_L + P_M + P_L$$

$$= 2P_L + P_M = -2\frac{(\mu - 1)}{R}$$

$\therefore$ focal length of the system

$$F = \frac{1}{P} = -\frac{R}{2(\mu - 1)} = -10\,cm$$

$$u = -15\,cm$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{F_{eq}}$$

$$\Rightarrow \frac{1}{v} - \frac{1}{15} = -\frac{1}{10}$$

$$\frac{1}{v} = \frac{1}{15} - \frac{1}{10} \Rightarrow v = -30 \text{ cm}$$

So, (B) is correct

7.(ABC) n for liquid = n for glass (yellow light)

n for liquid < n for glass (red light)

n for liquid $> n$ for glass (blue light)

8.(ABC) No. of maxima of interference in central maxima of diffraction is independent of λ.

9.(ABD) Analyse by using mirror formula

10.(ABCD) $Q = a\sqrt{2gh_1}$; $h_1 = \dfrac{Q^2}{a^2(2g)} = \dfrac{H}{2}$

$$\Rightarrow F = \rho\left(\sqrt{2g\frac{H}{4}}\right)^2 a = \frac{\rho g H a}{2}$$

$$\Rightarrow -A\frac{dh}{dt} = a\sqrt{2gh} : \int_0^t dt = \frac{-A}{a\sqrt{2g}}\int_{H/4}^{0}\frac{dh}{\sqrt{h}}$$

$$t = \frac{2A}{a\sqrt{2g}}\sqrt{\frac{H}{4}} = \frac{A\sqrt{H}}{a\sqrt{2g}}$$

If tap is not closed while bottom hole is closed and side hole is open then for water level to be steady

$$Q = a\sqrt{2gh(h - H/4)}$$

$$\Rightarrow \quad Q^2 = a^2\left(2g\left(h - \frac{H}{4}\right)\right) \Rightarrow a^2 gH = a^2 2g\left(h - \frac{H}{4}\right) \Rightarrow h = \frac{3H}{4}$$

PARAGRAPH FOR Q-11 & 12

11. (B) Since there is no shift in central maxima. Therefore, path difference introduced by the two sheets are equal i.e. $(\mu_1 - 1)\,t_1 = (\mu_2 - 1)\,t_2$

where μ_1 and μ_2 are refractive indices.

i.e. $\frac{t_1}{t_2} = \frac{(\mu_2 - 1)}{(\mu_1 - 1)} = \frac{(1.5-1)}{(1.25-1)} = \frac{0.5}{0.25} = 2$

12. (A) Path difference, $\Delta x = \frac{d^2}{2D}$, For first missing wavelength, $\frac{d^2}{2D} = \frac{\lambda}{2}$ or $\lambda = \frac{d^2}{D}$

SECTION-2

1.(3) Due to first refraction image will be formed at $\frac{4}{3}d$ from plane surface. This will act as an object for curved surface.

$$\frac{1}{v} + \frac{4/3}{\left(4 + \frac{4}{3}d\right)} = \frac{1 - \frac{4}{3}}{-4}$$

For image at infinity, $\Rightarrow \frac{4/3}{\infty} - \frac{1}{v} = \frac{4/3 - 1}{R}$ $\Rightarrow$ $v = -3R$

$$\Rightarrow -\frac{1}{3\times(4)} + \frac{4/3}{4\left(1+\frac{d}{3}\right)} = \frac{1}{12}\ ; \quad \frac{1}{3\left(1+\frac{d}{3}\right)} = \frac{1}{12} + \frac{1}{12} = \frac{1}{6} \Rightarrow 1 + \frac{d}{3} = 2$$

$d = 3cm$

2.(32) Given $2\Delta\ell_A = \Delta\ell_B$

$$\frac{2f}{A_a\gamma}\ell_A = \frac{F\ell_B}{16A_a\gamma} \quad ; \quad \frac{\ell_A}{\ell_B} = \frac{1}{32}$$

3.(36) Distance between two virtual images is $h = \sqrt{h_1 h_2} = 3mm$

(where $h_1 = 4.5mm$ and $h_2 = 2mm$)

Magnification in 1st case $= m_1 = \frac{4.5}{3} = \frac{v}{u} = \frac{l-x}{x}$ (where x is the object distance)

Magnification in 2nd case $= m_2 = \frac{2}{3} = \frac{x}{(l-x)}$

$$\Rightarrow \frac{3}{2} = \frac{x}{l-x} l = 150\,cm$$

$$\frac{1}{x} - \frac{1}{-(l-x)} = \frac{1}{f}$$

Solving $f = 36\,cm$

4.(2) $P_{heater} = P_{given\ to\ block} + P_{Loss\ to\ surroundings}$

$\therefore\ 500 = ms\dfrac{dT}{dt} + 0$

$500 = C\,2.5$

$\therefore\ C = 200\ J/°C$ (where is thermal capacity of the block)

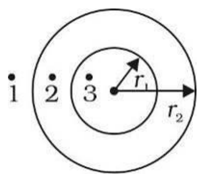
At 50°C, power loss to surroundings = 100 W

$0 = ms\dfrac{dT}{dt} + 100$

$\dfrac{dT}{dt} = -\dfrac{100}{200} = -0.5C°/s$

$n = 2$

5.(2) $P_2 - P_1 = \dfrac{4T}{r_2}$



$P_2 - P_0 = \dfrac{4T}{r_2}$ …(i)

$P_3 - P_2 = \dfrac{4T}{r_1}$ …(ii)

From equation (i) & (ii) $P_3 - P_0 = \dfrac{4T}{r_2} + \dfrac{4T}{r_1}$

$\dfrac{4T}{r_{eq}} = \dfrac{4T}{r_2} + \dfrac{4T}{r_1}$

$r_{eq} = \dfrac{r_1 r_2}{r_1 + r_2} = \dfrac{3\times 6}{3+6} = 2cm$

6.(16) $\tan i = \dfrac{4}{3}$

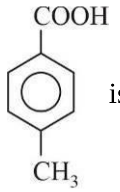
$i = 53° \Rightarrow r = 37°$

$D = 53° - 37° = 16$

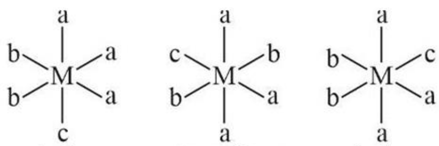
CHEMISTRY

SECTION-1

Single Choice Type

1.(B)  is less acidic due to (+H) effect of $-CH_3$ group

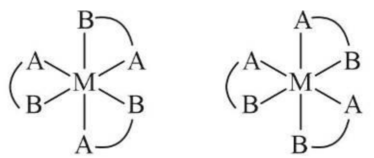
2.(D) I.



cis-form trans-form cis-form

$\Rightarrow$ All are optically inactive; No. of stereoisomers = 3

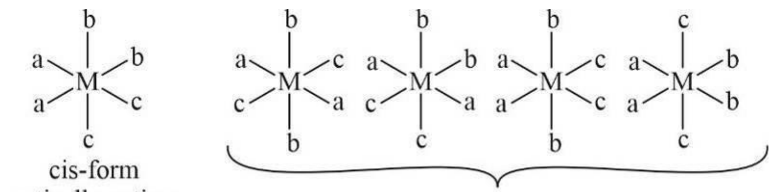
II.



cis-form optically active trans-form optically active

$\Rightarrow$ Both are optically active ; No. of stereoisomers = 4

III.



cis-form optically active

trans-form optically inactive

$\Rightarrow$ No. of stereoisomers = 6

3.(A) $Mn^{2+} + S_2O_8^{2-} \longrightarrow SO_4^{2-} + MnO_4^-$

$(meq)_{Mn^{2+}} = (meq)_{S_2O_8^{2-}}$

$5 \times 1 = 2 \times n$; $n = 2.5$

4.(C) $HAuBr_4 + 4HCl \rightarrow HAuCl_4 + 4HBr$; $\Delta H = (-28) - (-36.8) = 8.8$ kcal

$\therefore$ Percentage reaction $= \dfrac{0.44}{8.8} \times 100 = 5\%$

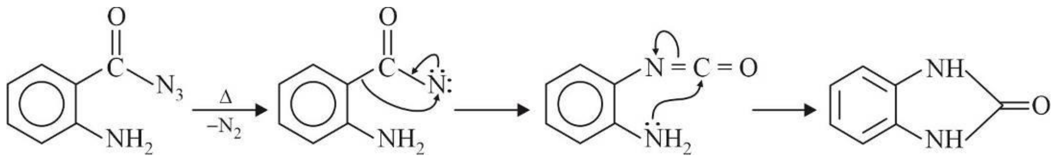
5.(B) For isothermal $\dfrac{dP}{dV} = -\left(\dfrac{P}{V}\right)$

For adiabatic $\dfrac{dP}{dV} = -\gamma\left[\dfrac{P}{V}\right]$

For P^2V = constant process $\dfrac{dP}{dV} = -\dfrac{1}{2}\left[\dfrac{P}{V}\right]$

Slope adiabatic > isothermal > $P^2V \Rightarrow$ So, I – 2, II – 1, III – 3

Multiple Correct Type

6.(BD) 

7.(BC) Irrespective of nature of ligand field, magnetic moment of octahedral complexes having metal ions in d^1, d^2, d^3, d^8, d^9 configuration, remains same.

Element may or may not form colourless complex.

8.(BC) Isoelectric point $= \dfrac{2+3.9}{2} = 2.95$

- At pH = 2.95 the molecule will exist as zwitter ion hence will not move towards cathode or anode.
- At pH = 1 the molecule will exist as cation hence will move towards cathode
- At pH = 10 and 12 the molecule will exist as anion hence will move towards anode

9.(ABCD)

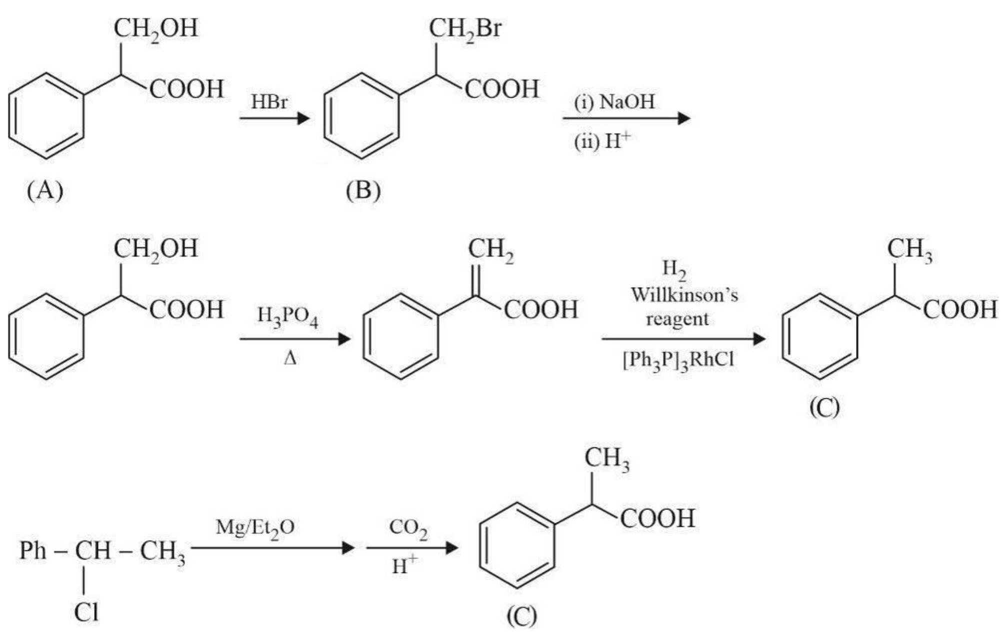
$$CH_3-CH_2-C\equiv N \xrightarrow{H_3O^+} CH_3-CH_2-COOH \xrightarrow[\text{(ii) } H_3O^+]{\text{(i) } Cl_2/P_{red}}$$

$$H_3C-\underset{}{\overset{\overset{\displaystyle Cl}{|}}{C}H}-COOH \xrightarrow{NH_3} H_3C-\overset{\overset{\displaystyle NH_2}{|}}{C}H-COOH$$

10.(CD) Fact based

PARAGRAPH FOR Q-11 & 12

11.(C) From the above discussion, structure of A satisfying all criteria of option 3. Option 3 when reacts with HBr will show retention as chiral carbon is not affected.

12.(B) 

SECTION-2

1.(4) (a) sp^3d^2 (b) d^2sp^3

(c) d^2sp^3 (d) d^2sp^3

(e) sp^3d^2 (f) d^2sp^3

(g) sp^3d^2

2.(75) For banana : $q = c \times 3.0$ …(i)

For benzoic acid : $\frac{0.305}{122} \times 800 = c \times 4.0$ …(ii)

From (i) and (ii), q = 1.5 kcal for 2.5 gm banana

$\therefore$ Heat obtained per banana $= \frac{1.5}{2.5} \times 125 = 75$ kcal

3.(3)
$$\begin{tikzcd} C(\text{graphite}) \arrow[r, "1\text{ bar}", "\Delta G^\circ = 5\text{ kJ/mol}"'] \arrow[d, "A"'] & C(\text{diamond}) \\ C(\text{graphite}) \arrow[r, "P\text{ bar}", "\Delta G^\circ = 0"'] & C(\text{diamond}) \arrow[u, "B"'] \end{tikzcd}$$

$G_{graphite} = G_{diamond}$ [at P bar]

For A : $dG = VdP - SdT$ (SdT = 0)

$(\Delta G)_A = V\Delta P = 5 \times 10^{-3}\ [P-1]$ bar L

For B : $(\Delta G)_B = V\Delta P = 3.33 \times 10^{-3}\ [1-P]$ bar L

$(\Delta G)_A + (\Delta G)_B = \Delta G^\circ$

$5 \times 10^{-3}[P-1] + 3.33 \times 10^{-3}[1-P] = \frac{5000}{100}$

$P[1.66 \times 10^{-3}] - 1.66 \times 10^{-3} = 50$

$P = \frac{50}{5} \times 3 \times 1000 = 30000 \text{ bar} \Rightarrow 3 \times 10^4 \text{ bar}$

4.(4) All monosaccharides whether aldose or ketose and all disaccharides except sucrose are reducing sugars. D(+) glucose & D(-) fructose form same osazone. D-mannose and D-galactose are not epimers. On changing pH secondary and tertiary structure gets destroyed while primary structure of proteins remain intact.

5.(2) $\underset{0.09}{RNH_2} + CHCl_3 + 3KOH \longrightarrow \underset{0.09}{RNC} + \underset{0.27}{3KCl} + 3H_2O$

$\underset{0.09}{RNH_2} + HNO_2 \longrightarrow ROH + \underset{0.09}{N_2} \uparrow$

Volume in litres at STP $= 0.09 \times 22.4 = 2.016 \cong 2$

6.(12) Compounds which are more acidic than $HC \equiv CH$ are (4) (6 – 16).

MATHEMATICS

SECTION-1

SINGLE CHOICE

1.(C) $2x^3ydy+(1-y^2)(x^2y^2+y^2-1)dx=0$

$$\frac{2y}{(1-y^2)^2}\frac{dy}{dx}+\frac{y^2}{1-y^2}\frac{1}{x}=\frac{1}{x^3}$$

Put $\dfrac{y^2}{1-y^2}=u$

$$\Rightarrow \frac{2y}{(1-y^2)^2}\frac{dy}{dx}=\frac{du}{dx}\Rightarrow \frac{du}{dx}+\frac{u}{x}=\frac{1}{x^3}$$

$$u.x=\int\frac{1}{x^2}dx+c \Rightarrow x^2y^2=(cx-1)(1-y^2)$$

2.(B) $f_n(x)=\begin{cases}\dfrac{x^n}{n!}\,;\,0\le x\le\dfrac{1}{2}\\[2mm] \dfrac{(1-x)^n}{n!}\,;\,\dfrac{1}{2}<x\le 1\end{cases}$

$$\Rightarrow I_n=\int_0^{1/2}\frac{x^n}{n!}dx+\int_{1/2}^{1}\frac{(1-x)^n}{n!}\,dx$$

$$=\frac{1}{(n+1)!}\left(\frac{1}{2}\right)^{n+1}+\frac{(1/2)^{n+1}}{(n+1)!}=2\left(\frac{(1/2)^{n+1}}{(n+1)!}\right)\Rightarrow \sum_{n=1}^{\infty}I_n=2\sqrt{e}-3$$

3.(B) $\dfrac{d}{dx}(x\sin x+\cos x)=x\cos x$ $\qquad f'(x)=x\cos x$

Let $f(x)=x\sin x+\cos x$ $\qquad f''(x)=-x\sin x+\cos x$

$$\int e^{f(x)}\left(xf'(x)+\frac{f''(x)}{(f'(x)^2)}\right)dx=\int xe^{f(x)}f'(x)dx+\int e^{f(x)}\cdot\frac{f'(x)}{(f'(x)^2)}dx$$

$$=xe^{f(x)}-\int e^{f(x)}dx+e^{f(x)}\frac{-1}{f'(x)}-\int e^{f(x)}\cdot f'(x)\left(\frac{-1}{f'(x)}\right)dx$$

$$=xe^{f(x)}-\frac{e^{f(x)}}{f'(x)}+C=e^{f(x)}\left(x-\frac{1}{f'(x)}\right)+C$$

$$=e^{x\sin x+\cos x}\left(x-\frac{1}{x\cos x}\right)+C$$

4.(D) $(f^3+g^2f)f'+f^2gg'=0$

$$\Rightarrow \left(\frac{1}{4}f^4+\frac{1}{2}f^2g^2\right)'=0\Rightarrow f^4+2f^2g^2=c$$

5.(A) $y^4 + xy - x^4 - x^3y^3 \ge 0$

$(y^3 + x)(y - x^3) \ge 0$

Thus, the two curves partition the circle $x^2 + y^2 \le 2$ into 4 regions of equal area and the inequality is satisfied by the two regions that cover the *y*-axis So, the area of the set is half the area of the circle.

ONE OR MORE THAN ONE CHOICE

6.(AB) $|z_1z_2| = \left|\frac{c}{a}\right|$ & $|z_1 + z_2| = \left|-\frac{b}{a}\right| = 1$

$(z_1 + z_2)(\bar{z}_1 + \bar{z}_2) = 1 \Rightarrow 2 + \bar{z}_1z_2 + z_1\bar{z}_2 = 1$

$\Rightarrow 2 + \frac{z_2}{z_1} + \frac{z_1}{z_2} = 1 \Rightarrow \frac{(z_1 + z_2)^2}{z_1z_2} = 1 \Rightarrow \frac{b^2}{a^2} = \frac{c}{a} \Rightarrow b^2 = ac$

Now, $z_2 = z_1e^{i\theta} \Rightarrow |z_1 + z_2| = |z_1|\left|1 + e^{i\theta}\right|$

$= 2\cos\frac{\theta}{2}\left|\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right|$

$|z_1 + z_2| = 2\cos\frac{\theta}{2} = 1 \Rightarrow \frac{\theta}{2} = \frac{\pi}{3} \Rightarrow \theta = \frac{2\pi}{3}$

Now, $PQ = |z_2 - z_1| = |z_1|\left|e^{i\theta} - 1\right| = \left|2\sin\frac{\theta}{2}\right|$ $\quad\therefore\ PQ = |z_2 - z_1| = \sqrt{3}$

7.(AB) $a_2 + b_2 = \int_0^{\pi/4} \ln(\sin x\cos x)\,dx$

$\Rightarrow a_2 + b_2 = \int_0^{\pi/4} \ln(\sin 2x)\,dx - \int_0^{\pi/4} \ln 2\,dx$

$2x = t \Rightarrow a_2 + b_2 = \frac{1}{2}\int_0^{\pi/2} \ln(\sin t)\,dt - \frac{\pi}{4}\ln 2$

Now, $a_1 = \int_0^{\pi/2} \ln(\sin t)\,dt = \int_0^{\pi/2} \ln(\cos t)\,dt$

$\Rightarrow 2a_1 = \int_0^{\pi/2} \ln(\sin t\cos t)\,dt \Rightarrow 2a_1 = \int_0^{\pi/2} \ln(\sin 2x)dx - \int_0^{\pi/2} \ln 2dx$

$= \frac{1}{2}\int_0^{\pi} \ln(\sin x)\,dx - \frac{\pi}{2}\ln 2$

$2a_1 = \int_0^{\pi/2} \ln(\sin x)dx - \frac{\pi}{2}\ln 2 \Rightarrow a_1 = -\frac{\pi}{2}\ln 2 \Rightarrow a_2 + b_2 = -\frac{\pi}{2}\ln 2$

8.(ACD) $(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}=(-x^2-4x+1)\vec{b}+\cos y\vec{c}$ and $\vec{a}\cdot\vec{b}=-\cos y$

Also, $\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}=6$

$-\cos y-x^2-4x+1=6$

$-x^2-4x-5=\cos y$

$-(x^2+4x+4)-1=\cos y$

$-(x+2)^2-1=\cos y$

Range of $-(x+2)^2-1=(-\infty,-1]$

Range of $\cos y=[-1,1]$

Hence solution is possible only when

L.H.S = R.H.S = -1

$\Rightarrow\ x=-2\ \&\ \cos y=-1$

$x=-2\ \Rightarrow\ y=(2n+1)\pi,\ n\in z$

$(x,y)\equiv(-2,(2n+1)\pi\ ;\ n\in z)$

(A) Put $x=-2$

$-4+y-\pi+4=0\Rightarrow\ y=\pi$

$(-2,\pi)$ satisfies

(B) $-2-y+2\pi+2=0$

If $y=2\pi$

$(-2,2\pi)$ does not satisfy

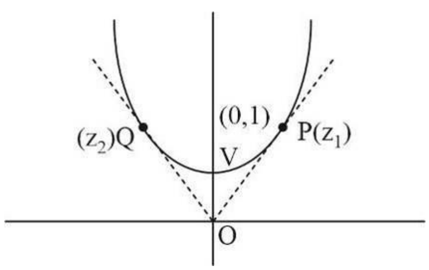
(C) $x=-2$ satisfy

(D) $-14-y+15\pi+14=0$

$(-2,15\pi)$ satisfies

9.(ACD) Put $z=x+iy$

$|ix-y+2|=y\ \Rightarrow\ (2-y)^2+x^2=y^2\ \Rightarrow\ x^2=4(y-1)$



$|z|_{\min}=\text{OV}=1$

$y-1=mx-m^2$ pass $(0,0)\ \Rightarrow\ m=\pm1$

$\Rightarrow\ \arg(z_1)=\dfrac{\pi}{4}\ \&\ \arg(z_2)=\dfrac{3\pi}{4}$

10.(AB) $$\int \frac{x^4+1}{x^6+1}dx = \int \frac{x^4-x^2+1}{x^6+1}dx + \int \frac{x^2}{x^6+1}dx$$

$$\int \frac{(x^4-x^2+1)}{(x^2+1)(x^4-x^2+1)}dx + \int \frac{x^2}{(x^3)^2+1}dx$$

$$= \tan^{-1}x + \frac{1}{3}\tan^{-1}x^3 + c = \frac{1}{3}\tan^{-1}\left(\frac{3x-3x^5}{1-3x^2-3x^4+x^6}\right) + c$$

COMPREHENSION TYPE

11.(A) $Y - y = m(X - x)$

Put $Y = 0$

$$X = x - \frac{y}{m}$$

Hence the quantity

$x - \frac{y}{m}$ is same for both the curves

$$x_1 - \frac{y_1}{m_1} = x_2 - \frac{y_2}{m_2} \text{ (but } x_1 = x_2\text{)}$$

$$\frac{y_1}{m_1} = \frac{y_2}{m_2} \text{ or } \frac{y_1}{y_2} = \frac{m_1}{m_2}$$

$$\frac{f(x)}{\int_{-\infty}^{x} f(t)\,dt} = \frac{f'(x)}{f(x)} \qquad \ldots\text{(i)}$$

Integrating equation (i), we get

$$\ln\left(\int_{-\infty}^{x} f(t)\,dt\right) = \ln(f(x)) + C$$

If $x = 0,\ \int_{-\infty}^{0} f(t)\,dt = \frac{1}{2}$ & $f(0) = 1 \Rightarrow C = \ln\left(\frac{1}{2}\right) = -\ln 2$

$$\therefore \ln\left(\int_{-\infty}^{x} f(t)\,dt\right) = \ln\left(\frac{f(x)}{2}\right)$$

$$\therefore f(x) = 2\int_{-\infty}^{x} f(t)\,dt \Rightarrow \frac{f(x)}{\int_{-\infty}^{x} f(t)\,dt} = 2 \qquad \ldots\text{(ii)}$$

Integrating equation (ii), we get $\ln\left(\int_{-\infty}^{x} f(t)\,dt\right) = 2x + C$

Put $x = 0 \Rightarrow C = -\log_e 2$

$$\ln\left(2\left(\int_{-\infty}^{x} f(t)\,dt\right)\right) = 2x$$

$$2\int_{-\infty}^{x} f(t)\,dt = e^{2x}$$

$2f(x) = 2e^{2x}$ ...(iii)

Differentiating both sides of equation (iii), we get

$f(x) = e^{2x}$

$$\lim_{x\to 0}\frac{f^2(x)-1}{x} = \lim_{x\to 0}\frac{e^{4x}-1}{x} = 4$$

12.(B) $f(x) = e^{2x}$

$f'(0) = 2$

Tangent at (0,1) is $y - 1 = 2x$

These lines meet the x-axis at $\left(-\frac{1}{2}, 0\right)$ & $(2, 0)$

$\therefore$ Area $= \frac{1}{2}\left(2+\frac{1}{2}\right)1 = \frac{5}{4}$

<u>NUMERIC VALUE TYPE</u>

1.(16) $|X| = PQ = \frac{|4x-3y|}{5}$

$|Y| = PR = \frac{|3x-4y|}{5}$

$\Rightarrow$ Given equation is $[|X|+[|Y|] = 3$

Symmetric about X-axis and Y-axis

For, $X, Y \ge 0$

$\Rightarrow$ Area = 4 sq. units

$\Rightarrow$ Total area enclosed $= 4\times 4 = 16$ sq. units

2.(37) $a_r = \frac{1}{(1+rx)(1+(r+1)x)} = \frac{1}{x(1+rx)} - \frac{1}{x(1+(r+1)x)}$

$$S_n(x) = \sum_{r=1}^{n}\frac{1}{x(1+rx)} - \frac{1}{x(1+(r+1)x)} = \frac{1}{x(1+x)} - \frac{1}{x(1+(n+1)x)}$$

$$S_n(x) = \frac{1}{x(1+x)} - \frac{1}{x(1+(n+1)x)} = \frac{1}{x}\left[\frac{(1+(n+1)x)-(1+x)}{(1+x)(1+(n+1)x)}\right]$$

$$S_n(x) = \frac{1}{x}\times\frac{nx}{(1+x)(1+(n+1)x)} = \frac{n}{(1+x)(1+(n+1)x)}$$

$$S_{50}\left(\frac{1}{3}\right) = \frac{50}{\left(1+\frac{1}{3}\right)\left(1+51\times\frac{1}{3}\right)} = \frac{50}{\frac{4}{3}\times 18} = \frac{25}{12}$$ $\therefore m+n = 37$

3.(8) $= 2\int_0^{\pi/2}\left[(\cos x - \cos^3 x) - \left(x^2 - \frac{\pi^2}{4}\right)\right]dx$

$= 2\int_0^{\pi/2}\left(\sin^2 x.\cos x - x^2 + \frac{\pi^2}{4}\right)dx$

$= 2\left(\frac{\sin^3 x}{3} - \frac{x^3}{3} + \frac{\pi^2}{4}x\Big|_0^{\pi/2}\right) = 2\left(\frac{1}{3} - \frac{\pi^3}{24} + \frac{\pi^3}{8}\right)$

$= \frac{2}{3} + \frac{\pi^3}{6}$

Here, $a = 6$

$xy = 6$ has 8 integral solutions, which are (2, 3), (3, 2), (1, 6), (6, 1), (-2, -3), (-3, -2), (-1, -6) and (–6, –1).

4.(1.50) $\frac{T_{n+1}}{2^{n+1}} = \frac{T_n}{2^n} + \frac{1}{2}\times 2^n$

$\Rightarrow \frac{T_2}{2^2} - \frac{T_1}{2} = \frac{1}{2}\times 2$

$\frac{T_3}{2^3} - \frac{T_2}{2^2} = \frac{1}{2}\times 2^2$

$\frac{T_n}{2^n} - \frac{T_{n-1}}{2^{n-1}} = \frac{1}{2}\times 2^{n-1}$

$\frac{T_n}{2^n} - \frac{T_1}{2} = \frac{1}{2}\frac{\left(2^{n-1}-1\right)\times 2}{2^{-1}} = 2^{n-1} - 1$

$\Rightarrow T_n = 2^{2n-1} - 2^{n-1} \Rightarrow S_n = \frac{2.4^n - 2 - 3.2^n + 3}{3} = \frac{2.4^n - 3.2^n + 1}{3}$

5.(20) $\vec{a} = \vec{b}\times\vec{c} + 2\vec{b}$ taking dot product with $\vec{b}$

$\vec{a}\cdot\vec{b} = 2\left|\vec{b}\right|^2 \Rightarrow \left|\vec{a}\right|\cdot\left|\vec{b}\right|\cos\theta = 2\left|\vec{b}\right|^2 \Rightarrow \cos\theta = \frac{4}{\left|\vec{a}\right|} \Rightarrow \theta = 0,$ as $\left|\vec{a}\right| = 4$

$\Rightarrow \vec{a} = 2\vec{b} \because \vec{b}\times\vec{c} = 0 \Rightarrow \vec{b} = \vec{c}$ *or* $\vec{b} = -\vec{c}$

So, $\left|2\vec{a} + \vec{b} + \vec{c}\right| \Rightarrow 3\left|\vec{a}\right| = 12$ or $2\left|\vec{a}\right| = 8$

$\therefore 12 + 8 = 20$

6.(5) $V_n = \int_0^1 e^x \cdot (x-x^2)^n = \int_0^1 e^x \left(x - x^2 + \frac{1}{4} - \frac{1}{4}\right)^n \cdot dx = \int_0^1 e^x \left(\frac{1}{4} - \left(x - \frac{1}{2}\right)^2\right)^n dx$

Put $x - \frac{1}{2} = t$

$$V_n = \int_{-1/2}^{1/2} \underset{\text{II}}{e^{t+\frac{1}{2}}} \underbrace{\left(\frac{1}{4} - t^2\right)^n}_{\text{I}} \cdot dt = \left(\frac{1}{4} - t^2\right)^n e^{t+\frac{1}{2}} \Bigg|_{-1/2}^{1/2} - n \int_{-1/2}^{1/2} \left(\frac{1}{4} - t^2\right)^{n-1} \times -2t \times e^{t+\frac{1}{2}} \cdot dt$$

$$V_n = 2n \int_{-1/2}^{1/2} \underset{\text{II}}{e^{t+\frac{1}{2}}} \underbrace{t\left(\frac{1}{4} - t^2\right)^n}_{\text{I}} \cdot dt$$

$$V_n = 2n \left[t\left(\frac{1}{4} - t^2\right)^{n-1} e^{t+\frac{1}{2}} \right]_{-1/2}^{1/2} - \int_{-1/2}^{1/2} e^{t+\frac{1}{2}} \left(1 \times \left(\frac{1}{4} - t^2\right)^{n-1} + (n-1) \times t \left(\frac{1}{4} - t^2\right)^{n-2} 2t\right)_{dt}$$

$$V_n = -2nV_{n-1} + 4n(n-1) \int_{-1/2}^{1/2} t^2 \left(\frac{1}{4} - t^2\right)^{n-2} \cdot e^{t+\frac{1}{2}} \cdot dt \quad \left[t^2 = -\left(\frac{1}{4} - t^2 - \frac{1}{4}\right)\right]$$

$$V_n = -2nV_{n-1} - 4n(n-1)V_{n-1} + n(n-1)V_{n-2}$$

$$V_n + (4n^2 - 2n)V_{n-1} - n(n-1)V_{n-2} = 0$$

$$k_1 = 4n^2 - 2n$$

$$k_2 = -n^2 + n$$

$$k_1 + k_2 = 3n^2 - n = n(3n-1)$$

Put $n = 100$

$$\frac{k_1 + k_2}{5980} = \frac{100 \times 299}{5980} = 5$$